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The choice of the mathematical method for prediction of electrochemical accumulator parameters value in power installations of space-rocket objects

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HIGHLIGHTS

- We reviewed several mathematical methods for prediction.
- We analyzed the use of methods for prediction of the performance of batteries.
- Long prediction of minimum voltage of nickel–cadmium battery was conducted.
- For prediction of batteries performance of space objects was selected Kalman filter.

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ABSTRACT

The review and analysis of several mathematical methods for prediction of electrochemical accumulator parameters are provided in the article: according to the mathematical expectation, the latest entry, a statistical prediction, Box–Jenkins model, decomposition Volta, ARMA, ARIMA and Kalman filter. The results of these methods for prediction of the electrochemical battery 22НКГ-4СК characteristics which is a part of spacecraft power plant of the “Mikrosputnik” type are given. Possible usage of these methods for long prediction of electrochemical accumulator characteristics on space-rocket objects power plants is showed.

Now nickel–cadmium accumulators, as a part of electrochemical battery system are widespread in power plants of space-rocket objects. The big resource, simplicity in operation, high mechanical durability and a wide temperature interval of operation are causes popularity of these electrochemical accumulators. However service life of space-rocket objects depends on a resource of electrochemical accumulators. At the same time, the actual service life of electrochemical accumulators is caused by a number of the degradation processes proceeding in them. Capacity loss, change and deterioration of charging and discharging characteristics, current increasing of the self-discharge and even complete failure of accumulators can be caused by these processes.

Thus, it’s a very important problem of accumulator’s state prediction based on known current values of their parameters and characteristics. This article covers the review of prediction methods characteristics of electrochemical accumulators and electrochemical batteries based on them for long term.

1. Problem formulation

Processes in accumulators which it is necessary to predict are described by dynamic series sequence of values of some quantities received during the certain time station [1]. A dynamic series includes two obligatory elements: time mark and value of an indicator of the row, received in several different ways and corresponding to the reference time mark.

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For prediction of a dynamic series it is necessary to determine the function dependence which describes a dynamic series adequately and which is called the prediction model [1]. The main aim of creation of the prediction model is receiving such model when the mean absolute deviation of true value from predicted value tends to the minimum [1]. After determining of prediction model it is necessary to calculate future values of a dynamic series and their confidence interval.

The basic requirements of mathematical methods in electrochemical accumulators of power plants prediction characteristics of space-rocket objects are formulated as [2]:

- adequacy it is the method that should show characteristics of electrochemical objects with the error not above the set;
- accuracy it is the characteristic of electrochemical accumulators calculated by means of mathematical method and it should possess high degree of coincidence to real accumulators characteristics values;
- universality it is the method that should display properties of the real electrochemical accumulator the most fully;
- efficiency it is the method that should yield result with the minimum expenses of machine memory and time, i.e. do a minimum of operations at one appeal to model.

The numbers of mathematical methods of prediction are being considered in the article and possibility of their application for prediction of electrochemical accumulator's characteristics in power plants of space-rocket objects will be analyzed. The prediction for nickel–cadmium battery 22HKГ-4CK (Sealed Ni–Cd-4SK) (consist of 22 electrochemical accumulators) working in a buffer mode as a part of power plant of the spacecraft "Mikrosputnik" type will be considered.

2. Review of mathematical methods of prediction

2.1. General concepts

Mathematical methods, showed below, are statistical method of prediction, therefore by them any characteristic of electrochemical accumulators that have defined set of statistic data could be predicted.

2.2. The prediction on the latest entry prognostic value

The prediction on the latest entry [3] prognostic value $\widehat{X}(t_0+\theta)$ is taken equal to the latest entry $X(t_0)$:

$$\widehat{X}(t_0+\theta) = X(t_0). \tag{1}$$

The predicted value does not depend on prediction time θ, and the background is presented only by one point – the latest entry $X(t_0)$, probabilistic characteristics are not considered at all.

The algorithm of a prediction is multiplication of value of the latest entry $X(t_0)$ to unit, i.e. does not demand performance in general any computing operations.

Error of a prediction $e(\theta)$ and variance of error σ_e^2 are equal:

$$e(\theta) = X(t_0+\theta) - X(t_0), \tag{2}$$

$$\sigma_e^2 = M\left\{[X(t_0+\theta) - X(t_0)]^2\right\} = \sigma_X^2 - 2R_X(\theta) + \sigma_X^2 = 2\left[\sigma_X^2 - R_X(\theta)\right]. \tag{3}$$

Variance of error increases from 0, at $\theta = 0$, before the doubled variance of process, at $\theta \to \infty$.

2.3. The prediction on the mathematical expectation

At a prediction on a mathematical expectation [3] prognostic value $\widehat{X}(t_0+\theta)$ is taken equal to a mathematical expectation of process m_X:

$$\widehat{X}(t_0+\theta) = m_X. \tag{4}$$

The predicted value does not depend on prediction time θ. Some data on properties of process are necessary: about its mathematical expectation. The algorithm of a prediction does not demand performance of any computing operations.

Error of a prediction $e(\theta)$ and variance of error σ_e^2 are equal:

$$e(\theta) = X(t_0+\theta) - m_X, \tag{5}$$

$$\sigma_e^2 = M\left\{[X(t_0+\theta) - m_X]^2\right\} = \sigma_X^2. \tag{6}$$

2.4. Methods of statistical prediction

2.4.1. The statistical prediction on one point

At a statistical prediction on one point [4] prognostic value $\widehat{X}(t_0+\theta)$ is taken equal to a conventional mathematical expectation of process at the time station $t_0+\theta$. The random variable of process at the time station t_0 is designated through the X, and at the time station $t_0+\theta$ through Y and the system of two random variables, the last value of prehistory and the predicted value (X, Y), is considered. According to predicting algorithm prognostic value $\widehat{X}(t_0+\theta)$ is equated to a conventional mathematical expectation of value Y at $X = X(t_0)$:

$$\widehat{X}(t_0+\theta) = m_{Y/X}. \tag{7}$$

Error of a prediction $e(\theta)$ and variance of error σ_e^2 are equal:

$$e(\theta) = X(t_0+\theta) - m_{Y/X}, \tag{8}$$

$$\sigma_e^2 = M\left\{\left[Y - m_{Y/X}\right]^2\right\} = \sigma_{Y/X}^2. \tag{9}$$

The mathematical expectation of a random variable of Y is equal:

$$m_{Y/X} = m_Y + r_{X,Y} \times \frac{\sigma_y}{\sigma_x} \times (x - m_x), \tag{10}$$

where $r_{X,Y}$ – is a coefficient of correlation between random variables X and Y; m_x and m_Y – are unconditional expectation of random variables X and Y $m_Y = m_x = m$;

σ_x and σ_y – are root-mean-square deviations of random variables X and Y $\sigma_Y = \sigma_x = \sigma$.

Subject to equation (10), equation (7) for calculation of prognostic value $\widehat{X}(t_0+\theta)$ will gain the following form:

$$\widehat{X}(t_0+\theta) = m + r_{X,Y} \times [X(t_0) - m]. \tag{11}$$

The variances of error σ_e^2 are equal:

$$\sigma_e^2 = \sigma^2\left[1 - \rho^2(\theta)\right]. \tag{12}$$

2.4.2. The statistical prediction on two and more points

At a statistical prediction on two and more points [4–6] system is formed by some random variables $X(t_2) = Z$, $X(t_0) = X$, $X(\theta) = Y$. Prognostic value $\widehat{X}(t_0+\theta)$ is taken equal to a conditional expectation $X(t_0+\theta)$ at $Z = X(t_2)$ and $X = X(t_0)$:

$$\widehat{X}(t_0+\theta) = m_{\frac{Y}{X}}Z. \tag{13}$$

In this case expression for calculation of prognostic value turns out very cumbersome:

$$\begin{aligned}\widehat{X}(t_0+\theta) = & \frac{\sigma_X^2 \times R(\theta) - R_X(\theta') \times R(\theta'-\theta)}{\sigma_X^4 - R^2(\theta'-\theta)} \times X(t_0) \\ & + \frac{\sigma_X^2 \times R(\theta') - R_X(\theta) \times R(\theta'-\theta)}{\sigma_X^4 - R^2(\theta'-\theta)} \times X(t_1) \\ & - m \times \left[\frac{R(\theta)+R(\theta')}{\sigma^2 + R(\theta'-\theta)} - 1\right].\end{aligned} \tag{14}$$

2.5. Mathematical model of Box–Jenkins

At a prediction stationary process by means of model of Box–Jenkins [5,7], autoregressive moving average ARMA (p,q) the abmodality x_t has the following form:

$$x_t = \sum_{i=1}^{p} \phi_i \cdot x_{t-j} + \varepsilon_t - \sum_{i=1}^{q} \theta_i \cdot \varepsilon_{t-i}. \tag{15}$$

where ε_t – is a random component, an error of prediction (white noise) on a step forward, with a mathematical expectation zero and variance $\sigma_e^2 > 0$; p – is an order of autoregression; q – is an order of moving average.

With the use of the lag operator L: $Lx_t = x_{t-1}$ this model can be written as follows:

$$\left(1 - \sum_{i=1}^{p} \phi_i \times L^i\right) y_t = \left(1 - \sum_{i=1}^{q} \theta_i \times L^i\right) \varepsilon_t \tag{16}$$

or

$$\phi(L) y_t = \theta(L) \varepsilon_t, \tag{17}$$

where $\varphi\,(L)$ – is a lag polynomial for autoregression; $\theta\,(L)$ – is a lag polynomial for a moving average.

2.6. Autoregressive moving average model

The equation of pure process of a reversibility of a moving average [5], without an autoregression component, in a form reminds decomposition Volta [7]. For each irreversible process of MA(q) there is the same reversible process of the same order on condition that root of the characteristic equation is not equal modulo to unit. For example, process MA(1) with $|\theta| > 1$ can be written down in the following look:

$$x_t = \varepsilon_t - \frac{1}{\theta} \varepsilon_{t-1}, \tag{18}$$

where $\varepsilon_t = (1 - \theta L/1 - (1/\theta)L)$ is white noise.

Having pro-differentiated equation of the ARMA process shifted on i of the periods forward, the following one can be received receive:

$$x_{t+1} = \sum_{i=1}^{p} \phi_i \cdot x_{t+i-j} + \varepsilon_{t+i} - \sum_{i=1}^{q} \theta_i \cdot \varepsilon_{t+i-j} \tag{19}$$

on ε_t:

$$\frac{\mathrm{d}x_{t+1}}{\mathrm{d}\varepsilon_t} = \sum_{i=1}^{p} \phi_i \cdot \frac{\mathrm{d}x_{t+i-j}}{\mathrm{d}\varepsilon_t} - \theta_i, \tag{20}$$

where $\theta_0 = -1$ and $\theta_i = 0$ at $j > q$.

Let's receive a recurrence formula for $\psi_i = (\mathrm{d}x_{t+1}/\mathrm{d}\varepsilon_t)$:

$$\psi_i = \sum_{i=1}^{p} \phi_i \times \psi_{i-j} - \theta_i. \tag{21}$$

At calculations for this formula it is necessary to consider that $\psi_0 = 1$ and $\psi_i = 0$ at $i < 0$.

At a prediction stationary process of ARMA [5,8,9] we will assume that all information on process x till T inclusive is available, i.e. information for a prediction coincides with full prehistory of process:

$$H_T = (x_T, x_{T-1}, \ldots). \tag{22}$$

If process of ARMA is reversible, on a basis (x_T, x_{T-1}, …) it is possible definitely determine error (ε_T, ε_{T-1}, …), using representation of ARMA model in the form of AR(∞):

$$\frac{\phi(L)}{\theta(L)} x_t = \pi(L) x_t = \varepsilon_t. \tag{23}$$

Having used the corresponding decomposition Volta (19) and equation (14) we will receive an optimum linear prediction for h of the periods forward, made at the moment of T:

$$\begin{aligned} x_T(h) &= \psi_h \times \varepsilon_T + \psi_{h+1} \times \varepsilon_{T-1} + \ldots = \sum_{i=0}^{\infty} \psi_{h+i} \times \varepsilon_{T-i} \overline{x}_{T+h} \\ &= \sum_{i=1}^{p} \phi_i \times \overline{x}_{T+h-i} + \overline{\varepsilon}_{T+h} - \sum_{i=1}^{q} \theta_i \times \overline{\varepsilon}_{T+h-i}. \end{aligned} \tag{24}$$

The prediction on the basis of equation (24) is the best linear prediction for any stationary reversible process of ARMA, at which error represents white noise. At fulfillment of conditions $E[\varepsilon_{T+i}|H_T] = 0$ such prediction will be equal to conditional expectation on prehistory of the predictand average, namely:

$$x_T(h) = E[x_{T+h}|H_T] \tag{25}$$

and consequently will be the best prognostic function. Thus values $\overline{x}_{T+i}$ and $\overline{\varepsilon}_{T+i}$ in equation (24) will be conditional relatively average of H_T.

At a prediction process of ARMA with the determined repressors [8,9], the determined regresses (a constant, a trend, etc.) can be included in ARMA in two different ways:

1. linear regression with ARMA process in a error:

$$x_t = X_t \beta + u_t, \tag{26}$$

where X_t – a line matrix of observation for regresses; β – a line matrix of coefficients of regression; u_t – the ARMA process which is set by equation (15) with replacement of x_t on u_t.

2. ARMAX process:

$$x_t = \sum_{i=1}^{p} \phi_i \times x_{t-i} + X_t \times \beta + \varepsilon_t - \sum_{i=1}^{q} \theta_t \times \varepsilon_{t-i}. \tag{27}$$

2.7. Autoregressive integrated moving average model

At a prediction of process of ARIMA [9,10] use two equivalent ways of prediction of the autoregressive integrated moving average model ARIMA (p,d,q):

$$\phi(L)(1-L)^d \times x_t = \theta(L)\varepsilon_t. \tag{28}$$

The first way is based on use of representation of ARIMA (p,d,q) in the form of ARMA ($p+d$,q):

$$\tilde{\phi}(L) \times x_t = \theta(L)\varepsilon_t, \tag{29}$$

where

$$\tilde{\phi}(L) = \phi(L)(1-L)^d. \tag{30}$$

The second way of prediction of the ARIMA model (p,d,q) consists in calculation of the necessary values for $w_t = (1-L)^d x_t$, i.e. for stationary process of ARMA (p, q), and receiving, on their basis, the corresponding index for x_T:

$$x_T(h) = x_T + \sum_{i=0}^{h} w_T(i). \tag{31}$$

2.8. Kalman filter

At a creation of estimates of a prediction and a filtration (Kalman filter) [9–12] model of a dynamic series is represented in terms of state space:

$$x_{k+1} = x_k + w_k, \tag{32}$$

$$x(t_0) = x_0, \tag{33}$$

$$y_k = x_k + \nu_k, \quad k = 1, 2, \ldots, N, \tag{34}$$

where x_k – is an ideal value of level of a studied dynamic series $\{y_k$, $k = 1, 2, \ldots, N\}$ at the instant of time from k to $k+1$ representing uncorrelated sequence with unknown average $E[w_k] = q$ and variance $E[w_k^2] = \sigma_w^2 = Q$; ν_k – is a chance sequence with a zero average and unknown variance $E[\nu_k^2] = \sigma_\nu^2 = R$.

The algorithm of creation estimates prediction and filtration includes the following stages:

- formation of sequence that is misalignments of the simplified filter:

$$\nu_k^{(2)} = y_k - y_{k-1}, \quad k = 2, 3, \ldots; \tag{35}$$

- estimation of average increase in level series q:

$$q_1(k|k) = q_1(k-1|k-1) + \frac{1}{k-1} \times \left(\nu_k^{(2)} - q_1(k-1|k-1)\right), \quad k = 2, 3, \ldots, \ q_1(1|1) = 0; \tag{36}$$

- construction of product sequences of the centered values of the misalignments:

$$y_k^w = 2\left(\nu_k^{(3)} - \frac{3}{2} \times q_1(k|k) \times \left(\nu_k^{(2)} - \frac{1}{2} \times q_1(k|k)\right)\right), \quad k = 3, 4, \ldots; \tag{37}$$

- estimation of noise variance in the model of the "dynamics" σ_w^2:

$$\sigma_{1w}^2(k|k) = \sigma_{1w}^2(k-1|k-1) + \frac{1}{k-2} \times \left[y_k^w - \sigma_{1w}^2(k-1|k-1)\right], \quad k = 3, 4, \ldots, \quad \sigma_{1w}^2(2|2) = 0; \tag{38}$$

- construction of value sequences of the misalignments:

$$y_k^\nu = \frac{1}{2} \times \left[\left(\nu_k^{(2)} - q_1(k|k)\right)^2 - \sigma_1(k|k)\right], \quad k = 2, 3, \ldots; \tag{39}$$

- estimation of noise variance of "measurer" σ^2:

$$\sigma_1^2(k|k) = \sigma_1^2(k-1|k-1) + \frac{1}{k-1} \times \left[y_k^\nu - \sigma_1(k-1|k-1)\right], \quad k = 2, 3, \ldots, \quad \sigma_1^2(1|1) = 0; \tag{40}$$

- construction of estimates sequence of level in series:

$$x_1(k+1|k) = x_1(k|k) + q_1(k|1), \tag{41}$$

$$x_1(1|1) = y_1, \tag{42}$$

$$x_1(k+1|k+1) = x_1(k+1|k) + K(k+1) \times (y(k+1) - x_1(k+1|k)). \tag{43}$$

Thus the coefficient of the filter calculated on the following equation is got:

$$K(k+1) = \frac{P(k+1|k)}{P(k+1|k) + \sigma_1^2(k+1|k+1)}, \tag{44}$$

where $P(k+1|k) = P(k|k) + \sigma_{1w}^2(k+1|k+1)$; $P(1|1) = \sigma_{1w}^2(1|1)$; $P(k+1|k+1) = (1 - K(k+1)) \times P(k+1|k)$.

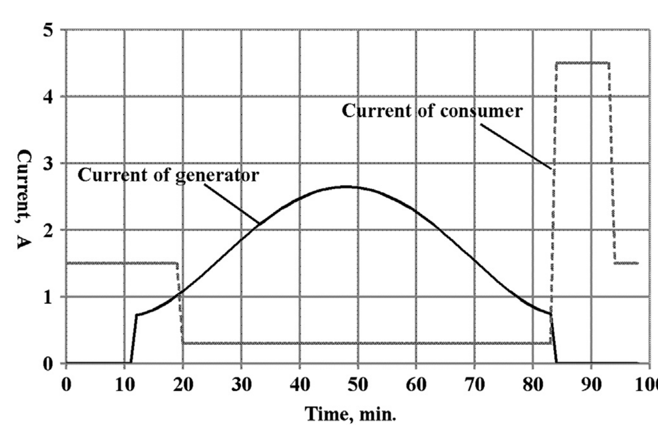

Fig. 1. Cycloramas of current of consumer and current of generator per one cycle.

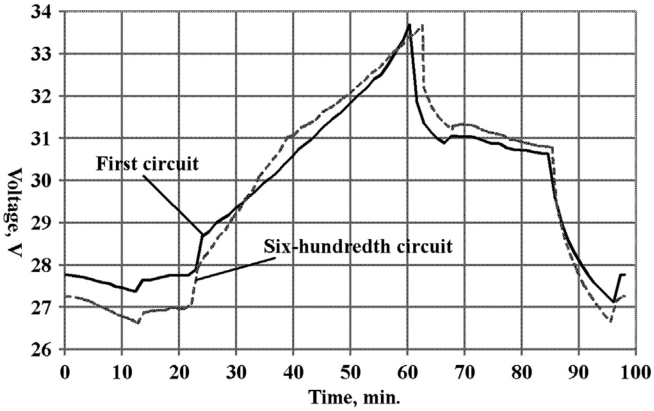

Fig. 2. Change of voltage for a cycle at the beginning and the end of the life tests.

3. The prediction examples of parameters values of the electrochemical battery with the use of the described methods

3.1. General concepts

For example regular the nickel–cadmium battery 22HKΓ-4CK working as a part of power plant in the spacecraft of "Mikro-sputnik" type, was carried out a long prediction of the minimum voltage per revolution. Ni–Cd battery 22HKΓ-4CK consists of 22 sealed Ni–Cd accumulators located in a metal frame. Battery available capacity is 4 A h, battery voltage – 1.37 V. Battery has two temperature-sensing elements and two threshold detectors. It's frame is metal with parameters of 380 × 320 × 70 mm, weight – 6.5 ± 0.05 kg. Battery designed for usage as a part of microsatellite power plant. For the given operating mode the main performance criterion of electrochemical battery is minimal voltage per revolution.

3.2. Life tests

The Ni–Cd 22HKΓ-4CK battery life testing has been occurred on special diagnostic tester for life testing (diagnostic tester is designed by authors) at the temperature of 25 ± 10 °C. The measured parameters were charging/discharging current, accumulators voltage and battery as a whole, temperature and charging/discharging length. Herewith the main control parameter was minimal battery voltage per one cycle.

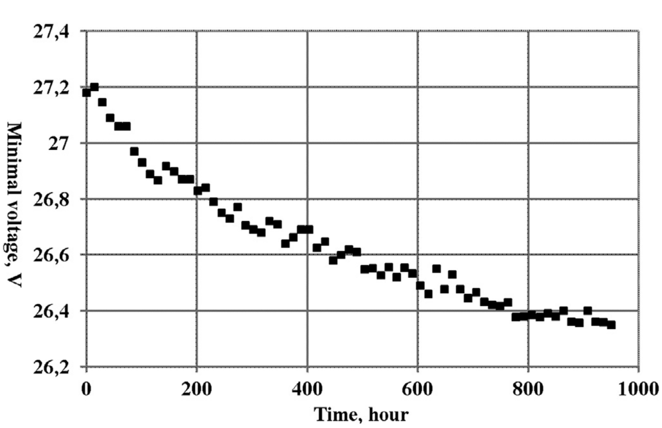

Fig. 3. Change of the minimal voltage of the electrochemical battery 22HKΓ-4CK during life tests.

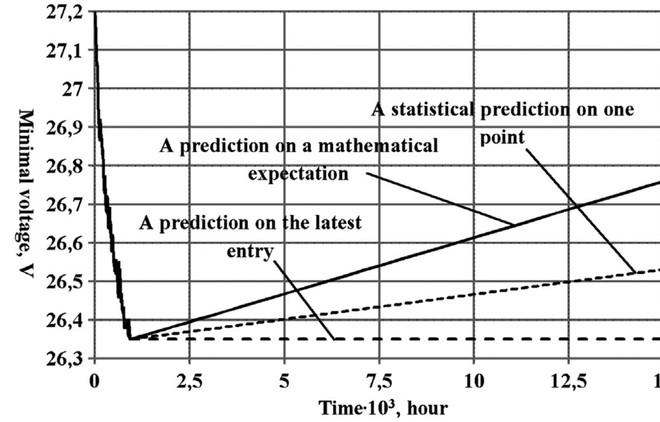

Fig. 4. Prediction of minimal voltage of the electrochemical battery 22HKΓ-4CK with the help of a prediction on a mathematical expectation, a statistical prediction on one point and a prediction on the latest entry.

When carrying out the life tests cycloramas of current of generator and current of consumer (Fig. 1) in all cycles, were considered to be constants.

But with the lapse of time the graph of electrochemical medium voltage variation will be modified. Minimal battery voltage decreases and the graph replaced to the right on the time axe. Fig. 2 shows such replacement per one cycle on the 1st and 600th cycles.

Basic data of life tests of these electrochemical batteries during six hundred cycles (980 h) on the basis of which the prediction is under construction are given on Fig. 3.

3.3. Minimal voltage prediction

In Figs. 4 and 5, results of a long prediction of change of minimum voltage of the battery 22HKΓ-4CK by means of the above-stated methods are given.

4. The analysis of mathematical methods of prediction

The analysis of the predictive data received as a result of above-mentioned methods usage, showed that a prediction method on the latest entry, a prediction method on a mathematical expectation and a statistical prediction at one point are not suitable for prediction of characteristics and parameters nickel–cadmium

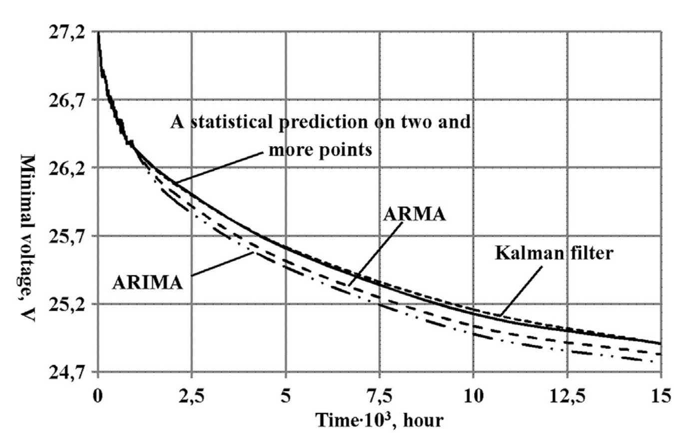

Fig. 5. Prediction of minimal voltage of the electrochemical battery 22HKΓ-4CK with the help of a statistical prediction at two and more points, ARMA, ARIMA and Kalman filter.

accumulators of power plants of space-rocket objects because of very low degree of accuracy and low adequacy.

According to such methods as a statistical prediction at two and more points, ARMA, ARIMA and Kalman filter, a high accuracy and adequacy are shown. The statistical prediction at two and more points appeared very bulky. A statistical prediction at two and more points, ARMA and ARIMA possess low degree of universality as well and demand high expenses of machine memory and time.

Thus, the most perspective method for prediction of electrochemical accumulators characteristics of space-rocket objects is Kalman filter possessing a high accuracy and adequacy, and also possessing universality and efficiency.

5. Conclusions

The review and the analysis of mathematical methods of prediction showed that there are a large number of the various methods now, possessing both advantages, and shortcomings. However for prediction of electrochemical accumulators parameters in power plants of space-rocket objects for long term of their operation, the only one method is Kalman filter, as the only method possesses rather high accuracy and adequacy, and it is universal and does not demand high expenses of machine memory and time with prediction values of electrochemical accumulators parameters in power installations of space-rocket objects for long term of their operation.

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